CALCULATE THE ECLIPSE

Brief Description

Students calculate the relative sizes of the Moon and the Sun in the sky and compare them to other moons in our Solar System. Primary and junior secondary students will use ratios for this exercise, and senior secondary students will use angular diameter (trigonometry).

School level: End of primary through secondary
Preparation time: None
Duration: 1 lesson or half-lesson or warm-up activity (easy to scaffold to your liking!)
Keywords: eclipse, Earth, Moon, Sun, Mars, Jupiter, solar eclipse, ratios, geometry, angular diameter, trigonometry

Educational Goals

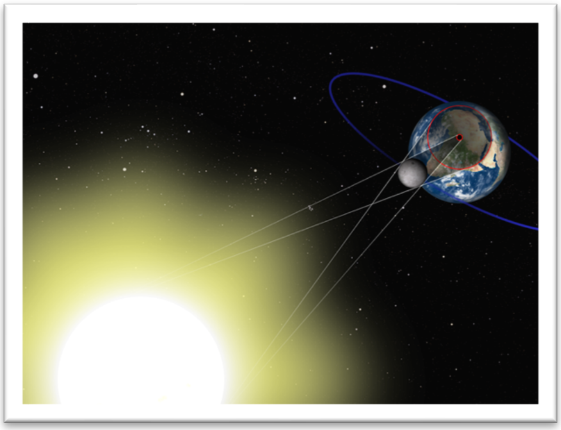
- ★ Apply math skills such as similar triangles, trigonometry, and ratios to astronomy
- ★ Understand why total solar eclipses on Earth are so special

Materials

- ❒ Worksheets
- ❒ Pencils, calculators

Introduction

A solar eclipse is a mesmerizing event that occurs when the Moon passes in between the Earth and the Sun in its orbit, causing the Sun to cast a shadow of the Moon onto the Earth. Solar eclipses are rare on Earth because the Earth, Moon, and Sun must be aligned perfectly so that the shadow of the Moon falls onto Earth and hits a populated land mass. Solar eclipses are even more rare when considering the Solar System at large – although almost all of our planets have moons, observers on other planets would not experience a solar eclipse in the same way that we do on Earth.



A total solar eclipse: the Moon passes between the Earth and the Sun in its orbit, causing the Sun to cast a shadow of the Moon onto Earth.

On Earth, the Moon and the Sun take up almost the same amount of space in the sky, so that during a total solar eclipse the Moon can almost completely "block out" the Sun. This is possible because even though the Sun is about 400 times further away from Earth than the Moon, it is also about 400 times larger than the Moon. On other planets, this ratio does not work out so nicely.

In this activity, we will calculate the relative sizes of the Sun and a moon in the sky for Earth, Mars, and Jupiter to better appreciate the rarity of this special event. Younger students can complete this activity using ratios and older students can use trigonometry.

Preparation

Print the attached worksheets.

Process

1. Introduce the concept of a solar eclipse to the class and explain why the relative sizes and positions of the Moon and the Sun result in both bodies having approximately the same size in the sky. You may consider using our Modelling Eclipses with Balls and/or our Modelling Eclipses to Scale activities.

2. Calculating the Eclipse with Ratios: Complete the attached worksheet. This can be done as a warm up activity or extended to an entire lesson. Depending on the ability of your students, consider only providing the values for diameter and distance and skipping questions a) and b) in each section so that students can work through creating the ratios needed by using similar triangles. Scaffold the activity to fit your needs!

3. Calculating the Eclipse with Angular Diameter (students must be familiar with trigonometry): Introduce the concept of angular diameter; following the worksheet, you can either provide students with the formula, derive it together as a class or in small groups, or allow students to derive it independently. Complete the attached worksheet. This is best suited as a warm-up activity or homework exercise, but can be extended to a full lesson if desired.

Taking it Further

Have students try both methods!

Students can extend their thinking to calculate the size of the Moon's shadow on Earth, using the Calculate the Eclipse Shadow activity below. It may be helpful to provide the answer to the first part of the activity to younger students and have them solve the second part only.

A fun extension is to calculate when the last total solar eclipse is expected to happen (adapted from Space Math @ NASA, values given are provided by NASA and Ask Astro):

1. Introduce elliptical orbits. The Moon changes its distance to Earth during its orbit, and Earth changes its distance to the Sun throughout the year. The furthest Earth can be from the Sun is 152,100,000 km. The closest the Moon can be to Earth is 363,300 km. The Moon is also moving away from Earth at a rate of 3.8 cm/year.

2. Discuss these questions with your students or add them to the worksheets:
    a. What is the furthest the Moon could be from Earth to match the smallest size of the Sun in our sky? (Answer: ~377,643 km)
    b. How much further away from Earth will the Moon? (Answer: ~14,343 km)
    c. When will this occur? (Answer: In ~377 million years! The true expected timeframe is ~600 million years, discuss with your students why there is a discrepancy, e.g. exact vs approximate or average values)

The calculations in these activities also provide a great opportunity to discuss significant digits or figures and scientific notation.

CALCULATING THE ECLIPSE WITH RATIOS

A solar eclipse is a mesmerizing event that occurs when the Moon passes in between the Earth and the Sun in its orbit, causing the Sun to cast a shadow of the Moon onto the Earth.

On Earth, the Moon and the Sun take up almost the same amount of space in the sky, so that during a total solar eclipse the Moon can almost completely "block out" the Sun. This is special to Earth – on other planets in our solar system, the moon(s) may be too small in the sky to fully block out the Sun or too big and block out the corona (the Sun's atmosphere) as well.

An image of the 2017 total solar eclipse.
Credit: Rémi Boucher

In this activity, we will use ratios to calculate the relative sizes of a moon and the Sun in the sky for Earth, Mars, and Jupiter. All measurements given are average values provided by the Canadian Space Agency and NASA.

Earth

Measurements:

- Moon diameter: 3476 km
- Moon distance from Earth: 384,400 km
- Sun diameter: 1,400,000 km
- Sun distance from Earth: 150,000,000 km

a) What is the ratio of the Moon's distance from Earth to its diameter?

b) What is the ratio of the Sun's distance from Earth to its diameter?

c) Why does the Moon fully "block out" the Sun during a solar eclipse from Earth?

Mars

Mars has two moons: Phobos ("fear") and Deimos ("panic"). Let's see what a solar eclipse looks like on Mars when Phobos passes between Mars and the Sun.

Measurements:

- Phobos diameter: 22 km
- Phobos distance from Mars: 6000 km
- Sun diameter: 1,400,000 km
- Sun distance from Mars: 227,939,200 km

a) What is the ratio of Phobos's distance from Mars to its diameter?

b) What is the ratio of the Sun's distance from Mars to its diameter?

c) Which is bigger in the sky, Phobos or the Sun? What would a solar eclipse look like on Mars?

d) Take it further: What would Phobos' diameter need to be to appear as the same size as the Sun in the sky from Mars?

Jupiter

According to NASA, Jupiter has 95 moons that we know about so far! Jupiter's largest moon is Ganymede. Let's see what a solar eclipse looks like on Jupiter when Ganymede passes between Jupiter and the Sun.

Measurements:

- Ganymede diameter: 5262 km
- Ganymede distance from Jupiter: 1,070,000 km
- Sun diameter: 1,400,000 km
- Sun distance from Jupiter: 778,600,000 km

a) What is the ratio of Ganymede's distance from Jupiter to its diameter?

b) What is the ratio of the Sun's distance from Jupiter to its diameter?

c) Which is bigger in the sky, Ganymede or the Sun? What would a solar eclipse look like on Jupiter?

d) Take it further: What would Ganymede's diameter need to be to appear as the same size as the Sun in the sky from Jupiter?

Answers

Earth:

a) Moon ratio: ~110
b) Sun ratio: ~107
c) eclipse: Moon can fully block out Sun, but not the corona. They are about the same size in the sky.

Mars:

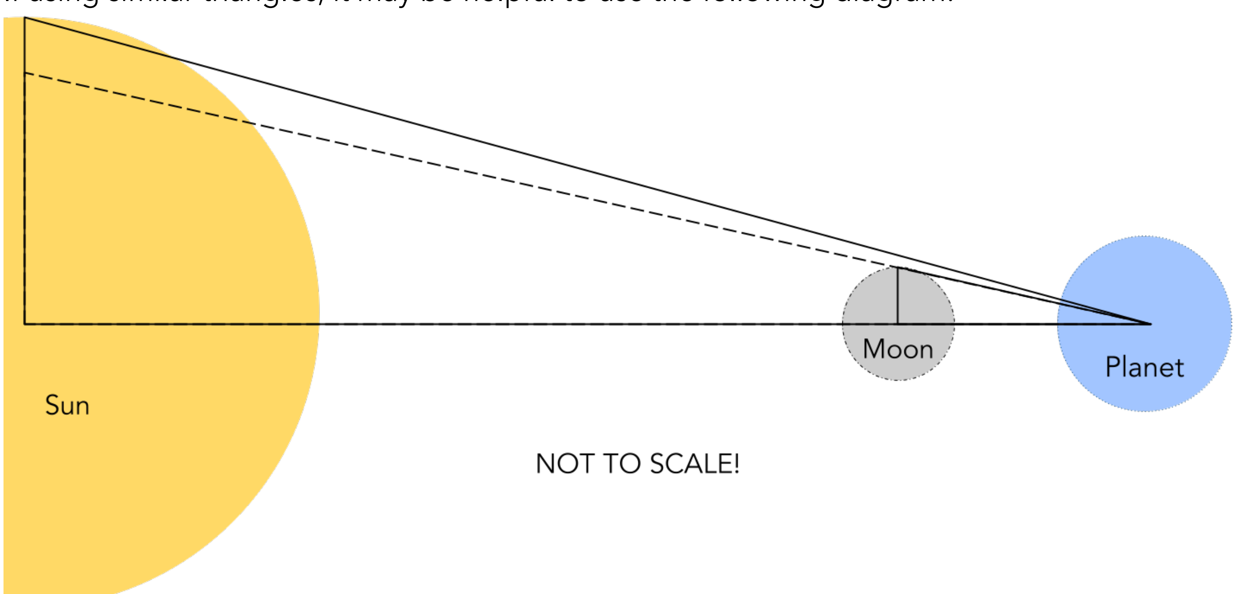
a) Phobos ratio: ~273
b) Sun ratio: ~162
c) eclipse: Phobos is too small! It is a little more than half as wide in the sky as the Sun
d) Calculate the ratio of the distance between Mars and the Sun to the distance between Mars and Phobos (~37,990). Then, divide the Sun's diameter by this number to get Phobos' necessary diameter to create a total solar eclipse (~37 km).

Jupiter:

a) Ganymede ratio: ~203
b) Sun ratio: ~556
c) eclipse: Ganymede is too big! It is almost 3 times as wide in the sky as the Sun
d) Calculate the ratio of the distance between Jupiter and the Sun to the distance between Jupiter and Ganymede (~728). Then, divide the Sun's diameter by this number to get Ganymede's necessary diameter to create a total solar eclipse (~1923 km).

Note: ratios may be inverses of what is presented here.

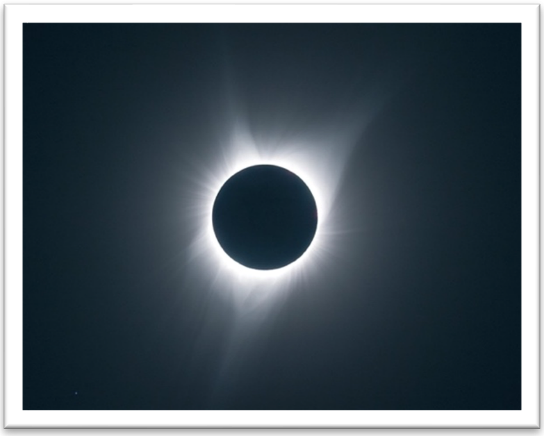
If using similar triangles, it may be helpful to use the following diagram:



CALCULATING THE ECLIPSE WITH ANGULAR DIAMETER

A solar eclipse is a mesmerizing event that occurs when the Moon passes in between the Earth and the Sun in its orbit, causing the Sun to cast a shadow of the Moon onto the Earth.

On Earth, the Moon and the Sun take up almost the same amount of space in the sky, so that during a total solar eclipse the Moon can almost completely "block out" the Sun. This is special to Earth – on other planets in our solar system, the moon(s) may be too small in the sky to fully block out the Sun or too big and block out the corona (the Sun's atmosphere) as well.



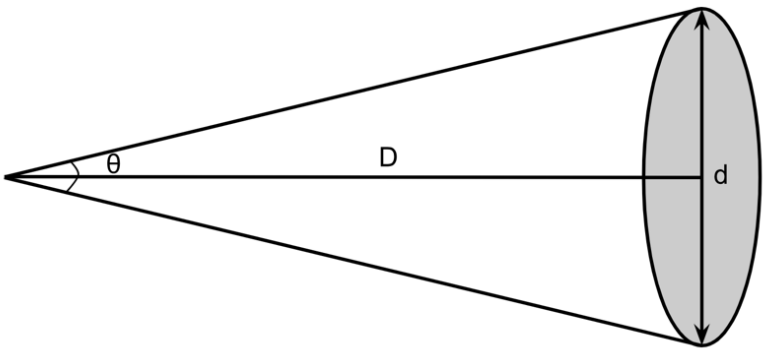
An image of the 2017 total solar eclipse.
Credit: Rémi Boucher

In this activity, we will use angular diameter to calculate the relative sizes of a moon and the Sun in the sky for Earth, Mars, and Jupiter. All measurements given are average values provided by the Canadian Space Agency and NASA.

Angular Diameter

Angular diameter is what astronomers use to measure how big an object appears in the sky. If astronomers know how far away an object is from Earth (which they can figure out from looking at what wavelengths or colours of light the object is giving off), they can use angular diameter to figure out how big the object really is. In this activity, we will do the reverse: we will use the diameters and distances of the Sun and moons to their respective planets to calculate how big each object is in the sky.

Let's derive a formula for angular diameter. Use the following diagram and your knowledge of right triangles.



d = diameter of object

D = distance to object

θ = angular diameter

Earth

Measurements:

- Moon diameter: 3476 km
- Moon distance from Earth: 384,400 km
- Sun diameter: 1.4×10^6 km
- Sun distance from Earth: 150×10^6 km

a) What is the Moon's angular diameter as seen from Earth?

b) What is the Sun's angular diameter as seen from Earth?

c) Why does the Moon fully "block out" the Sun during a total solar eclipse from Earth?

Mars

Mars has two moons: Phobos ("fear") and Deimos ("panic"). Let's see what a solar eclipse looks like on Mars when Phobos passes between Mars and the Sun.

Measurements:

- Phobos diameter: 22 km
- Phobos distance from Mars: 6000 km
- Sun diameter: 1.4×10^6 km
- Sun distance from Mars: 227.9392×10^6 km

a) What is Phobos's angular diameter as seen from Mars?

b) What is the Sun's angular diameter as seen from Mars?

c) Which is bigger in the sky, Phobos or the Sun? What would a solar eclipse look like on Mars?

d) Take it further: What would Phobos' diameter need to be to appear as the same size as the Sun in the sky from Mars?

Jupiter

According to NASA, Jupiter has 95 moons that we know about so far! Jupiter's largest moon is Ganymede. Let's see what a solar eclipse looks like on Jupiter when Ganymede passes between Jupiter and the Sun.

Measurements:

- Ganymede diameter: 5262 km
- Ganymede distance from Jupiter: 1.07×10^6 km
- Sun diameter: 1.4×10^6 km
- Sun distance from Jupiter: 778.6×10^6 km

a) What is Ganymede's angular diameter as seen from Jupiter?

b) What is the Sun's angular diameter as seen from Jupiter?

c) Which is bigger in the sky, Ganymede or the Sun? What would a solar eclipse look like on Jupiter?

d) Take it further: What would Ganymede's diameter need to be to appear as the same size as the Sun in the sky from Jupiter?

Answers

Angular diameter formula:

$$2\arctan\left(\frac{d}{2D}\right)$$

Earth:

a) Moon angular diameter: ~0.518°
b) Sun angular diameter: ~0.535°
c) Eclipse: Moon can fully block out Sun, but not the corona. They are about the same size in the sky.

Mars:

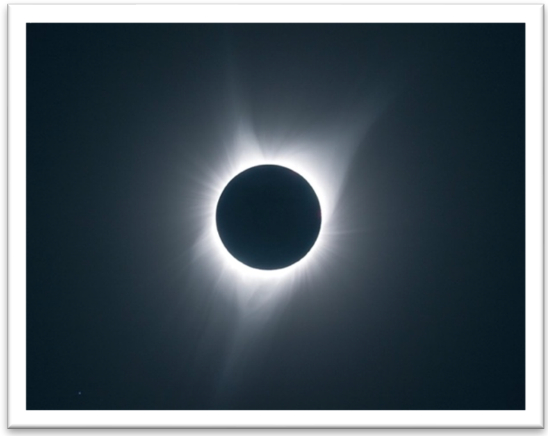
a) Phobos angular diameter: ~0.210°
b) Sun angular diameter: ~0.352°
c) Eclipse: Phobos is too small! It is a little more than half as wide in the sky as the Sun
d) ~37 km

Jupiter:

a) Ganymede angular diameter: ~0.282°
b) Sun angular diameter: ~0.103°
c) Eclipse: Ganymede is too big! It is almost three times as wide in the sky as the Sun
d) ~1923 km

CALCULATING THE ECLIPSE SHADOW

(Adapted from "Epic Eclipse" by NASA)



An image of the 2017 total solar eclipse.
Credit: Rémi Boucher

A solar eclipse is a mesmerizing event that occurs when the Moon passes in between the Earth and the Sun in its orbit, causing the Sun to cast a shadow of the Moon onto the Earth.

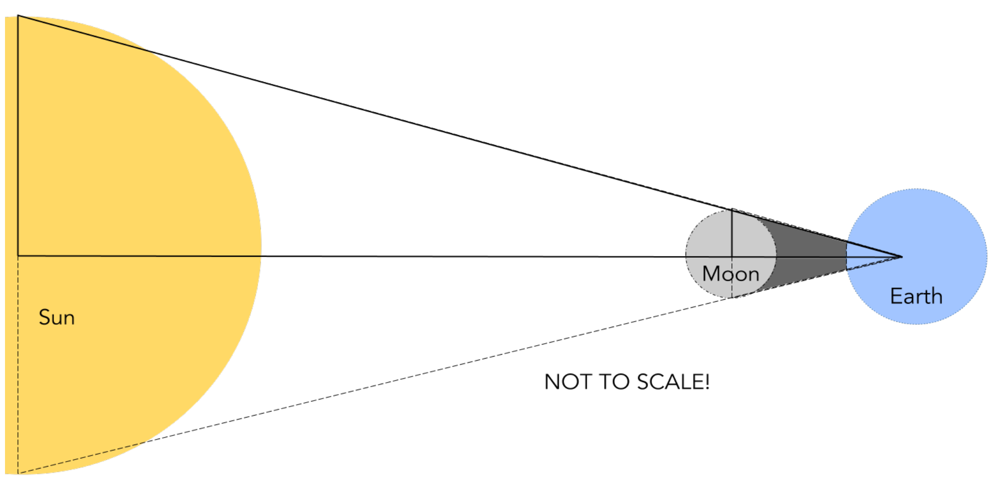
During a total solar eclipse, the Moon completely blocks out the Sun in a small part of its shadow. The size of this shadow determines how much of the Earth's surface will be in totality.

In this activity, we will use similar triangles to find the area of the Moon's shadow during a total solar eclipse. Note that this activity requires multi-step problem solving. All measurements given are average values provided by the Canadian Space Agency and NASA.

Estimating the shadow cone

The Moon's shadow on the Earth during a total solar eclipse will usually be an ellipse, but we will make the assumption in this activity that it is a circle. We will also ignore aspects like topography and assume that the Moon and the Earth are smooth spheres.

When the Sun casts light on the Moon, the Moon's shadow will create a cone that intersects with the surface of the Earth. The vertex of the cone isn't necessarily at the centre of the Earth. Use the below diagram and known distances to determine the full length of the shadow cone cast by the Moon if it wasn't blocked by the Earth. Consider using similar triangles, and note that the given distances are between the outermost surfaces of the objects, not their centres.



Measurements:

- Sun Diameter: 1,400,000 km
- Moon Diameter: 3476 km
- Earth Diameter: 12,742 km
- Sun distance from Earth: 150,000,000 km
- Minimum Moon distance from Earth: 363,300 km

a) Find the full length of the shadow cone.

Area of the shadow

Knowing the length of the shadow cone from the Moon to its vertex (if it wasn't blocked by the Earth), we can find the area of the shadow at this intersection using similar triangles. Try redrawing the Moon and Earth part of the above diagram to help you!

Draw the system!

a) Find the portion of the Moon's shadow that is blocked by the Earth.

b) Using similar triangles, find the radius of the Moon's shadow on the surface of the Earth.

c) What is the area of the Moon's shadow during a total solar eclipse?

Answers

You and your students may notice that we take the smallest distance between the Earth and the Moon for this activity, instead of the average distance between them. This is to ensure that the shadow cone intersects the Earth and the Moon actually casts a shadow on the Earth in this generic activity. Specific distances on specific days when total solar eclipses have happened or will happen can also be used, and are encouraged since you may then be able to compare your results for the area of the shadow with what was observed or is expected.

Estimating the shadow cone: a) ~376,536 km

- Reasoning: We can create similar right triangles using the diagram in the activity, one that is from the cone vertex to the centre of the Sun (bigger triangle) and one that is from the cone vertex to the centre of the Moon (smaller triangle). The length of the bigger triangle is the length of the smaller triangle plus the distance from the Sun to the Moon plus the radius of the Moon and the radius of the Sun, and its height is the radius of the Sun. You can find the distance between the Sun and the Moon during a total solar eclipse by subtracting the distance between the Moon and the Earth from the distance between the Earth and the Sun. The height of the smaller triangle is the radius of the Moon, and its length is unknown. Solve for the length of the smaller triangle.

Area of the shadow: a) ~11,692 km

- Reasoning: There are multiple ways to find this value. One option is to notice that the length of the shadow cone can be broken down into known distances, specifically the radius of the Moon, the distance between the Moon and the Earth, and the length of the cone blocked by the Earth, which is what we want to find. For other system orientations, the length of the shadow may extend past the centre of the Earth. Then the length of the shadow intersected by the Earth may be found by subtracting the length of the cone from the distance between the centres of the Earth and the Moon.

Area of the shadow: b) ~54 km

- Reasoning: The shadow cone from the Moon to its vertex and from where it intersects the surface the Earth to its vertex are similar triangles. The bigger triangle's height is the radius of the Moon and we solved for the length in the first question. The smaller triangle's length is what we found in the previous question, and we're solving for its height.

Area of the shadow: c) ~9161 km^2

- Find the area of the circle using $A = \pi r^2$